Five Year Integrated M.Sc. 2022
Subject: Mathematics (Theory of Optimization)
Course Code: MT-3-5-3
Full Marks.-80
Time: 4 Hrs.

Attempt **Question No. 1** and any **three** from the rest.

1. (i) Write down the mathematical form of a general Linear Programming Problem.
(ii) What is feasible solution?
(iii) What do you mean by unbounded solution?
(iv) A company has two grades of inspectors, I and II, who are to be assigned for a quality control inspection. It is required that at least 2000 pieces be inspected per 8 hour day. Grade I inspectors can check pieces at the rate 50 per hour with an accuracy of 97%. Grade II inspectors can check pieces at the rate 40 per hour with an accuracy of 95%. The wage rate of Grade I inspectors is Rs. 4.50 per hour and that of Grade II is 2.50 per hour. Each time an error is made by an inspector, the cost to the company is one rupee. The company has available, for the inspection job, 10 grade I and 5 grade II inspectors. Formulate the problem to minimize the total cost of inspection. 5+2+3+10=20

2. (i) Solve the following linear programming problem by drawing a diagram:

$$\begin{gathered}\text{Maximize } 3x_1 + 2x_2 \\ \text{subject to} \\ -x_1 + 3x_2 \leq 12 \\ x_1 + x_2 \leq 8 \\ 2x_1 - x_2 \leq 10 \\ x_1, x_2 \geq 0\end{gathered}$$

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(ii) Define basic feasible solution. What is hyperplane? Show that a hyperplane is a convex set. 10+(2+2+6)=20

3. (i) Explain Simplex method in detail.
(ii) Solve the following LPP problem using the simplex algorithm:
Maximize $3x_1 + x_2 + 3x_3$
subject to
$2x_1 + x_2 + x_3 \leq 2$
$x_1 + 2x_2 + 3x_3 \leq 5$
$2x_1 + 2x_2 + x_3 \leq 6$

$x_1, x_2, x_3 \geq 0$.
(iii) Explain in detail, degeneracy in Simplex method. 5+10+5=20

4. (i) Convert the following LPP to standard form and solve:
Maximize $z = 3x_1 - 2x_2 + 4x_3$
subject to
$x_1 + 2x_2 + x_3 \leq 8$
$2x_1 - x_2 + x_3 \geq 2$
$4x_1 - 2x_2 - 3x_3 = -6$
$x_1, x_2, x_3 \geq 0$.
(ii) Construct the dual of the following and solve both the primal and the dual:
Maximize $2x_1 + x_2$
subject to
$-x_1 + 2x_2 \leq 2$
$x_1 + x_2 \leq 4$
$x_1 \leq 3$
$x_1, x_2 \geq 0$. 10+10=20

5. (i) Write a general formulation of the problem when primal is in standard form and hence construct the dual of the problem.
Maximize $z = 3x_1 + 17x_2 + 9x_3$
subject to
$x_1 - x_2 + x_3 \geq 3$
$-3x_1 + 2x_3 \leq 1$
$x_1, x_2, x_3 \geq 0$.

(ii) Consider the problem of assigning four operators to four machines. The assignment costs in rupees are given here. Operator I can not be assigned to machine III and operator 3 can not be assigned to machine IV. Find the optimal cost of assignment.

	I	II	III	IV
1	5	5	-	2
2	7	4	2	3
3	9	3	5	-
4	7	2	6	7

10+10=20

6. (i) Explain the Matrix Minima Method for finding a basic feasible solution of a transport problem

(ii) Solve the following transportation problem:

	D_1	D_2	D_3	D_4	a_i
O_1	10	7	3	6	3
O_2	1	6	8	3	5
O_3	7	4	5	3	7
b_j	3	2	6	4	

10+10=20